

Cryogenic design of a high- T_c SQUID-based heart scanner cooled by small Stirling cryocoolers

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A heart scanner based on high- T_c SQUIDs is currently under development at the University of Twente. It is intended to be used in standard clinical environments without a magnetically shielded room. In order to make the application simple to use, the SQUIDs will be cooled by small cryocoolers, thus realizing a turnkey apparatus. The aimed field resolution is $50 \text{ fT}_{\text{RMS}} \text{ Hz}^{-1/2}$ in a measuring band of 0.1–100 Hz. The mechanical cooler interference is reduced by incorporating two coolers and operating them in counter phase. The magnetic cooler interference is reduced by positioning the coolers and the SQUIDs in a coplanar arrangement, and by separating the SQUIDs from the cold tips with a solid conducting thermal interface. A design is presented in which a temperature of 55 K is expected with a cool-down time of less than 1 h. © 1997 Elsevier Science Limited

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Superconducting QUantum Interference Devices (SQUIDs) are the most sensitive magnetic flux-to-voltage converting sensors. At present, their main application lies in the field of biomagnetic research where multichannel low- T_c dc-SQUID based magnetometer systems are used. These systems are usually cooled by liquid helium and operated in magnetically shielded rooms to obtain an extremely low-noise measuring environment. They are expensive, require helium refills and cannot be transported in a simple manner.

Because of the higher operating temperature, a much more flexible magnetometer system can be realized with high- T_c SQUIDs. The workhorse of current high- T_c superconducting electronics, $\text{YBa}_2\text{Cu}_3\text{O}_7$, with its critical temperature (T_c) of 92 K opened the possibility of operating SQUIDs in liquid nitrogen at 77 K. Also, small-scale turnkey cryocoolers, originally developed for cooling infrared sensors, are available and are very reliable¹. The advantages of cryocoolers compared to a liquid-nitrogen cryostat are the lower operating temperature (which gives a potentially better SQUID performance²), the turnkey operation (no refills required), and the possibility of operating the system in all directions.

In this paper the design of a heart scanner is presented that can be equipped with up to 25 high- T_c SQUIDs cooled by two small Stirling-type cryocoolers. In the next section the design goals and constraints are reviewed, in which special attention is paid to the heart signal and the required

sensor resolution. After that, the specific coolers that are used in this project are discussed. Because of the interference at the SQUID positions arising from the cryocoolers, a separation in space or time through a thermal interface is demanded. A separate section deals with possible thermal interfaces which can transfer heat from the SQUID unit to the coolers, and with special measures that have to be taken to reduce the magnetic and mechanical cooler interference. Then, the thermodynamic aspects of the cooling system for the heart scanner using a conductive-strip thermal interface are considered. The paper concludes with the actual design of the heart scanner.

Design goals and constraints

We aim to realize a high- T_c SQUID-based heart scanner than can be used in clinical environments in a flexible manner. That means that it should be transportable and simple to use. Furthermore, it should be cheap compared to multichannel low- T_c systems that typically cost 1.5 million US dollars. As a consequence, a magnetically shielded room is out of the question and the suppression of the environmental noise is a major challenge. The suppression of the environmental magnetic noise has been studied but is, however, beyond the scope of this paper^{3,4}.

Heart signal and sensor resolution

The QRS-complex signal strength in the magnetic human heart signal (magnetocardiogram, MCG) of a healthy adult is typically 10–100 pT peak-to-peak³. The spectral information of the heart signal is mainly contained in the frequency band 0.1–40 Hz³. However, in order to detect anomalies in the MCG, higher frequencies, up to 100 Hz, should also be considered. High- T_c SQUIDs can be used to adequately record MCGs provided that their noise levels are 50 fT_{RMS} Hz^{-1/2} or lower³.

In the heart scanner project we use high- T_c SQUIDs with bicrystal grain boundary junctions. Different coupling schemes were studied: monolithic flux-transformer coupled, flip-chip coupled and directly coupled (or inductively shunted)⁵. A directly coupled SQUID shunted with a single pick-up loop requires only one layer of high- T_c material and is, therefore, relatively simple to manufacture. So far, we have made these latter magnetometers with a 10 mm × 10 mm sensing loop — using the whole substrate area — and a noise level of roughly 100 fT_{RMS} Hz^{-1/2}. Figure 1 shows a typical MCG measured by an inductively shunted high- T_c SQUID that was cooled by liquid nitrogen and operated in the magnetically shielded room of the Biomagnetic Centre Twente⁶. As a next step in the SQUID design, larger substrates of 20 mm × 20 mm can be used to lower the intrinsic field noise to 50 fT_{RMS} Hz^{-1/2}.

Requirements

As regards the cryocoolers, the requirements concern the operating temperature, the temperature stability and the magnetic and mechanical interferences.

The higher operating temperature of high- T_c compared to low- T_c systems is a big advantage with respect to flexibility and costs. As an option, liquid nitrogen boiling at 77 K can be used instead of liquid helium (nitrogen is one order of magnitude cheaper and has a factor of 50 higher volumetric heat of evaporation). However, an operating temperature lower than the boiling temperature of liquid nitrogen gives a potentially better field resolution², and may relax the required temperature stability (see below). A rule of thumb for proper SQUID performance prescribes an operating temperature of 0.6 T_c or lower². Because YBa₂Cu₃O₇ has a T_c of 92 K, we aim at an operating temperature of 60 K or (preferably) lower. Such a low operating temperature can be realized with small-scale cryocoolers, as is indicated in the following sections.

The required temperature stability strongly depends on the type of high- T_c SQUID that is used. The most sensitive to temperature fluctuations are inductively shunted SQUIDs. The effective sensing area of these SQUIDs is dramatically affected by temperature due to changes in the temperature-dependent London penetration depth. The higher the operating temperature and the higher the external background magnetic field, the more stringent is the required stability. To reduce various noise contributions, we plan to compensate for the earth's magnetic field by a factor of 40 dB using a compensation coil^{3,7}. In that case, in order to achieve the required resolution, the inductively shunted SQUIDs manufactured in our group demand a stability of 0.3 mK_{pp} for harmonic fluctuations when operating the SQUIDs at 77 K (assuming a band width of 100 Hz)⁸. As in a liquid nitrogen bath temperature fluctuations of a few tenths of a kelvin may be expected⁹, thermal buffering via relatively large heat capacities will be required. In our design, cryocoolers will be used to cool the SQUIDs and a temperature in the range of 55 K is attainable. Operating the inductively shunted SQUIDs at 55 K, the temperature fluctuations should be limited to 2 mK_{pp} (again with a factor of 40 dB reduction in the earth's magnetic field). Flux-transformer coupled SQUIDs, integrated as well as 'flip-chip' configurations, give values for the stability requirements that are roughly one order of magnitude less severe^{3,8}. A stable temperature can be realized by actively controlling the cryocooler input power¹⁰.

Because dc-SQUIDs are nearly non-dissipative in the stationary state, the cooling power for 25 SQUIDs is mainly used to compensate for the thermal load, which is predominantly caused by the heat conduction from the supporting material and the leads to the sensors. Because the heat capacity of the sensors is smaller compared to the capacity of the rest of the cooling system, the latter will determine the cool-down time. This cool-down time, the weight and the heat capacity of the rest of the system that needs to be cooled, and the cooling power interrelate. For example, assuming that the rest of the system is fabricated from copper or silicon, about 0.1 MJ kg⁻¹ is required to cool from 300 K to 50 K, if the rest of the system is supposed to be thermally insulated from the environment. Then, with an estimated weight of 0.5 kg and a cool-down time of 2 h, for a cryocooler's cooling power $\dot{q}_{cc-p}$ that is assumed to be proportional to the operating temperature T , we find: $\dot{q}_{cc-p} = a(T - T_{min})$ with $a = 0.056$ W K⁻¹ and $T_{min} = 50$ K. Clearly, at the start of a cool-down a larger cooling power is available with cryocoolers than close to the stationary state.

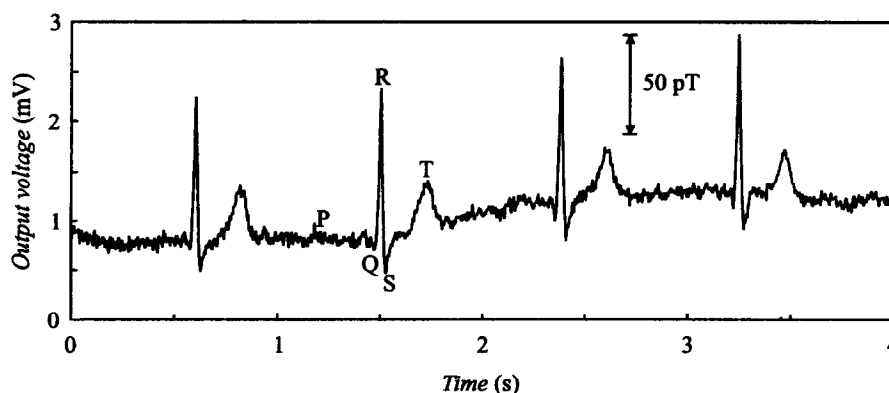


Figure 1 Typical magnetocardiogram recorded by an inductively shunted high- T_c dc-SQUID⁶ at 77 K, cooled by liquid nitrogen, with a field resolution of about 100 fT_{RMS} Hz^{-1/2} in the magnetically shielded room of the Biomagnetic Centre Twente

The cryocoolers that we plan to use operate at the power line frequency of 50 Hz and their magnetic interference will, therefore, mainly be of this frequency. The magnetic noise present in clinical environments shows 50 Hz peaks of typically 10–50 nT_{RMS}³; therefore, we consider a 50 Hz noise contribution of the coolers below 10 nT_{RMS} as acceptable. Peaks at other frequencies should be smaller, roughly below 1 pT_{RMS}. This value results from the required resolution of 50 fT_{RMS} Hz^{-1/2} in our measuring band width (100 Hz)³.

Concerning mechanical vibrations due to the coolers, the main contribution again will be at 50 Hz. However, noise may also arise at lower frequencies due to band widening effects and non-linear mechanical coupling. The main effect will be through axial tilting of the SQUIDS in the external magnetic field. Vibrations in the earth's magnetic field can give 1 μ T per degree of tilt. Taking into account a field compensation factor of 40 dB, the acceptable magnetic interference can be directly transformed to acceptable mechanical interference. At the power line frequency a sensor tilt of 1 degree_{RMS} is allowed and at the other frequencies 0.1 millidegrees_{RMS}. The requirements are summarized in Table 1.

Cryocoolers to be used in the heart scanner

Small cryocoolers have been developed especially for cooling infrared detectors mainly in military applications (Stirling type as well as Linde–Hampson type with Joule–Thomson nozzle)¹. These reliable and turnkey coolers are commercially available with refrigeration power ranging from typically 0.2–2 W, specified at 80 K with an environmental temperature around 20°C¹. Both types of small cooler meet the requirements for the cooling system of the heart scanner, but we concluded that for the heart scanner Stirling cryocoolers are preferred over Linde–Hampson coolers³. This is because Stirling coolers are small, light-weight, reliable, efficient, simple, maintenance-free (once sealed for their working lifetime), have a low specific power consumption, low mechanical and electromagnetic noise levels and can be manufactured at a reasonable price¹.

Stirling-type coolers are manufactured, among others, by Signaal Usfa (Eindhoven, The Netherlands)¹¹ (see Figure 2). In these coolers, schematically depicted in Figure 3, two compressor pistons are driven by linear motors. The motion of each piston is generated by a time-varying magnetic field of a coil which interacts with a permanent magnetic field around the coil; the oscillating field



Figure 2 Photograph of Signaal Usfa UP Stirling cryocoolers. Referring to the compressors from left to right: types 7057, 7058 and 7056; respective cooling powers are 0.75 W, 1.5 W and 0.5 W; compressor size¹¹ about 12 cm (Courtesy of Signaal Usfa)

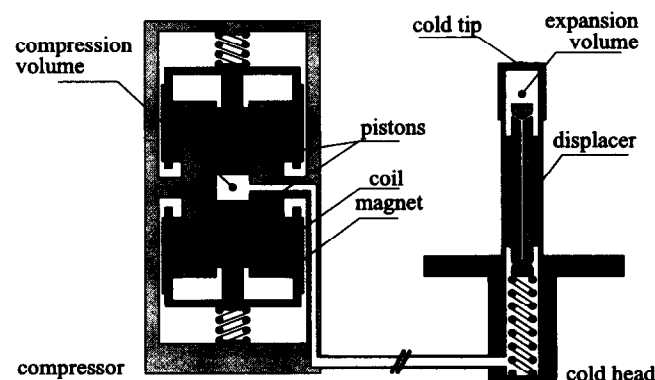


Figure 3 Cross-section of compressor and displacer units of dual opposed UP 7058 split-Stirling cryocooler¹¹ (Courtesy of Signaal Usfa)

is caused by an alternating current through the coil. The coil is connected to the piston. In this way a (helium) gas-pressure wave is produced which acts on the spring-loaded displacer in the cold heat. This displacer moves in such a way that the expansion of the gas and thus the refrigeration mainly takes place in the expansion space at the tip of the cold finger, whereas the compression occurs in the compression space between the compressor pistons and the warmer end of the displacer. The mass of the compressor piston/coil assembly, in combination with the mechanical spring and gas forces, creates a mass–spring system that is

Table 1 Requirements for heat scanner with respect to cryocoolers³

Parameter	Requirement	
Operating temperature (K)		<60
Temperature stability for inductively shunted SQUIDS at 55 K ^a	Harmonic fluctuations (mK _{pp})	<2
Mass of the sensors to be cooled (kg)		<0.05
Stationary cooling power for the sensors (W)		>0.2
Cool-down time for the sensors (h)		<1–2
Magnetic interference at the sensors	White noise (fT _{RMS} Hz ^{-1/2})	<50
	Power line (50 Hz) (nT _{RMS})	<10
	Other peaks (pT _{RMS})	<1
Mechanical interference: sensor tilt ^a	Power line (50 Hz) (deg _{RMS})	<1
	Other peaks (millideg _{RMS})	<0.1

^aIncluding a 40 dB earth's field compensation

tuned to the drive frequency of 50 Hz, in order to provide a good practical approximation to the ideal phase relationship between the displacer and the pistons. Two dual opposed pistons are used in each single compressor in order to balance the axial forces, thus reducing the mechanical vibrations. The Signaal Usfa coolers were described more extensively elsewhere^{3,11}.

Figure 4 shows the cooling power $\dot{q}_{cc}$ of the Signaal Usfa UP 7058 cooler as a function of temperature^{11,12}. Each 7058 type has a cooling power of 1.5 W at 80 K if operated at room temperature. The required input power is 55 W in that case. For the heart scanner we plan to use a set of two coolers (type UP 7058), firstly because two coolers can be mounted back to back so that the axial forces of the displacer units can be balanced and secondly to have sufficient cooling power.

In order to achieve the demanded resolution, attention has to be paid to the reduction in the mechanical and magnetic interferences due to the coolers.

Cooler interference

When a cryocooler is applied for cooling a SQUID magnetometer it will generate noise in several ways. Firstly, by magnetic interference: direct magnetic noise due to the compressor currents or due to the vibration of magnetic cooler components. Secondly, by mechanical interference: indirect magnetic noise due to vibrations of the cooler causing movements of the SQUID magnetometer in the environmental field, including the static magnetic field of the cooler. Thirdly, noise due to the fact that metal is used in the vicinity of the SQUID pick-up loops: magnetic noise arising from the thermal motion of electrons in the metal ('Johnson noise') and distortion of environmental fields by eddy currents creating relatively strong gradients. And finally, by thermal interference: thermal fluctuations affect the London penetration depth, changing the effective sensing area of the SQUIDs⁸. In this paper, attention is paid to the magnetic, mechanical and thermal interference. The Johnson noise can be evaluated according to references 13 and 14, whereas the field distortion can be estimated by the

worst case in which the metallic components are considered to be superconductive. Then the effect can be estimated on the basis of reference 15.

If the SQUIDs were mounted directly on the cold fingers of the coolers, the required resolution for measuring human heart signals could not be attained due to the magnetic interference arising from the coolers¹⁶. Therefore, to reduce the cryocoolers' interference the coolers have to be separated from the SQUID unit, which can be realized in time or in space. In the *time-separation* concept a latent cold reservoir is necessary which from time to time has to be re-cooled. Then, the noise due to the cooler is limited to that due to the metal used in the vicinity of the SQUIDs. Proper design and realization, possibly with the adjustment of the factory-standard cooler itself by applying non-magnetic materials in the cooler, can reduce this noise contribution. This time-separation method was investigated by Kaiser *et al.*¹⁷.

The advantage of *space-separated* cooling systems is the non-interrupted operation of the SQUID magnetometer as well as of the cryocooler. The first reason is a clear advantage with respect to the biomagnetic measuring facility, whereas the second is relevant with respect to wear of the cryocooler and, therefore, its lifetime. Because of the possibility of non-interrupted use of the SQUID-based magnetometer system, we chose to apply the space-separated concept. Moreover, in this space-separation method it is expected that the magnetic and mechanical noise due to the cooler can be reduced substantially by a proper, sufficiently nearby positioning of the cooler(s) with respect to the SQUIDs. This can be achieved by the positioning of the compressors in a so-called back-to-back configuration combined with the application of active or passive noise suppression techniques and, possibly, the use of a μ -metal shield.

A space-separated system can be realized in three possible ways: using a gas circulation¹⁸, a heat pipe¹⁹ or a conductive strip. These possible thermal interfaces for the heart scanner have been considered elsewhere^{3,20}. In all space-separated systems the necessary distance between the cooler and the SQUIDs is determined by the noise produced by the cooler. If the length can be kept sufficiently restricted in view of the magnetic interference from the cooler, a conductive strip is a simple and adequate method for cooling. This cooling method is described in a following section of the paper.

The reduction of the magnetic and mechanical interference due to the two selected Stirling coolers is considered in the following part of this section.

Reduction of magnetic cryocooler interference

The magnetic field generated by the 50 Hz currents in the compressor coils can be described as dipolar with the magnetic moment located in the centre of the compressor, pointing along its axis. The magnetic moment of the Signaal Usfa UP 7058 compressor was measured²¹ to be about $0.5 \text{ A m}^2_{\text{RMS}}$. Because the field is dipolar, it is possible to position the coolers with respect to the SQUID magnetometers in such a way that a minimum pick-up of the compressor field is established. SQUID magnetometers are sensitive to magnetic fields which have a component normal to the plane of their sensing coils. Therefore, the coolers should be positioned in such a way that their resulting magnetic field at the SQUIDs lies in the plane of

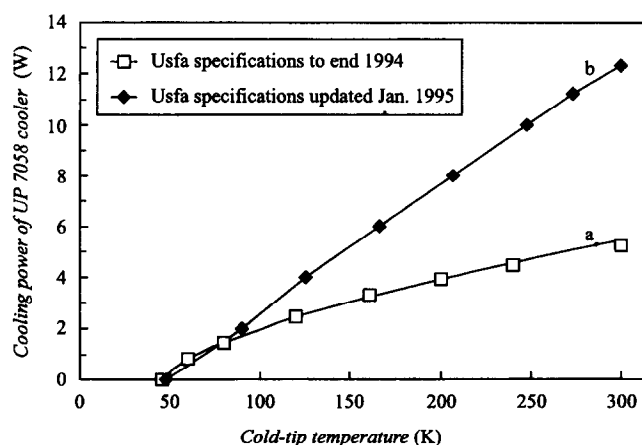


Figure 4 Cooling power $\dot{q}_{cc}$ of UP 7058 cooler as a function of cold-tip temperature T_c as specified (at an input power of 55 W and an environmental temperature of 20°C) by Signaal Usfa^{11,12}. Curve a has been applied for the optimization and comparison of the two cooling configurations, whereas curve b has been used for the simulation of the measurements in the test set-up and of the final cryogenic design of the heart scanner

the sensing coils. Suppose these coils are in the XY -plane, so that they are sensitive to B_z (see the inset in Figure 5). In this case, a possible orientation for the two coolers is along the y -axis with a distance b in between. In this balanced configuration, the two dipoles are directed oppositely to each other, leading to a resulting 50 Hz field along the x -axis. In Figure 5, the amplitude B_d of this field as a function of the distance x is given for two dipoles each with strength $m_d = 0.5 \text{ A m}_{\text{RMS}}^2$ and separated by a base distance b of 0.15 m between the centres of the compressors. At a distance of 0.5 m to the SQUIDS, B_d equals about $3.6 \times 10^{-7} \text{ T}_{\text{RMS}}$. We expect that we can realize a coplanar arrangement within an accuracy of 1° . Therefore, the SQUIDS sense a maximum normal component of about $6 \text{ nT}_{\text{RMS}}$, which is below the acceptable level of $10 \text{ nT}_{\text{RMS}}$ ^{3,7}.

Moreover, we plan to reduce the compressor-current noise contributions even further by means of the following method. In the fixed configuration of the coolers and SQUIDS, the transfers of the compressor currents to the detected fields in all SQUIDS are measured. Then, during the heart measurements, the compressor currents are measured and, because the transfers are known, the compressor-current field contributions can simply be subtracted from all SQUID outputs.

Another contribution to the 50 Hz noise may arise from movements of magnetic cooler components. These concern the compressor magnet and the slightly magnetic regenerator material inside the displacer that oscillates with a 3 mm amplitude intrinsic to the cooler operation. However, we have estimated that in our design this contribution is far below the acceptable level^{3,22}, even more so if vibration-reduction techniques are applied (see below).

As a result of these considerations, the SQUIDS will be separated from the compressors by 0.5 m, in which case the distance of the SQUIDS to the cold heads is 0.3 m.

Reduction of mechanical cryocooler interference

A UP 7058 cooler consists of a compressor and a displacer module interconnected by a rather stiff pipe. As mentioned before, each compressor comprises two pistons that move

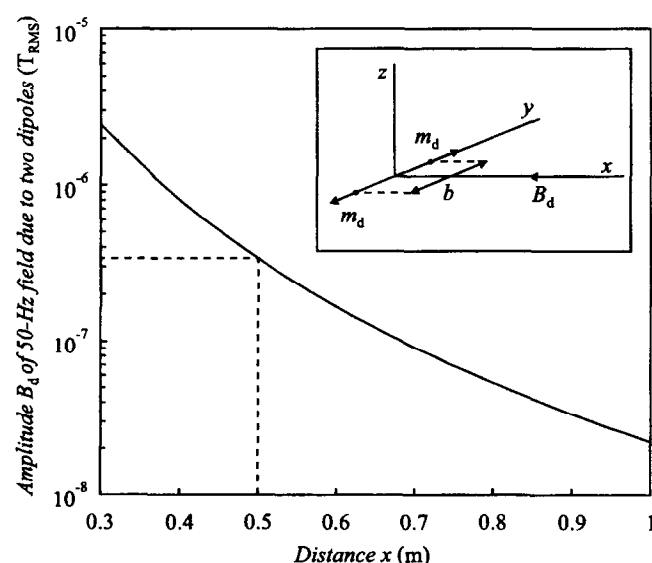


Figure 5 Amplitude B_d of 50 Hz magnetic field due to two dipoles as a function of distance x . The inset shows the position of the dipoles with respect to the plane of the pick-up coils of the SQUIDS

in opposite directions to balance the axial forces. In standard coolers the two compressor coils are connected in parallel. A small imbalance remains, resulting in vibration peaks of about 1 N corresponding to accelerations of roughly 1 m s^{-2} . Our cryocoolers were manufactured with separate current connections for the two compressor coils. By tuning the compressor currents we were able to improve the balance and thus to reduce the 50 Hz peak by a factor of 50, while the 100 Hz and the 150 Hz peaks were also reduced (see Figure 6). The experiments were performed in a test set-up in which the compressor was mounted on thin flexible plates, thus allowing relatively large displacements in the axial direction²². The accelerations were detected by an acceleration transducer bonded to the compressor.

As a further result the tuning, and thus the balance of the axial forces in the compressor, appeared to be very stable in time, and also independent of the cold-tip temperature. Therefore, we plan to use the following concept for the heart scanner. Each of the two compressors is first characterized in the flexible test set-up. The optimum ratio of the amplitudes and the optimum phase difference of the compressor-coil currents are determined. After that, both coolers (compressors and cold heads) are rigidly mounted in a back-to-back configuration on a supporting structure that also carries the SQUIDS. The compressor-coil currents are controlled by means of a digital signal processor board according to the predetermined conditions. In this configuration the remaining vibrations of the supporting structure are predominantly caused by the imbalance in the axial forces of the two separate displacers. These vibrations will be detected and fed back to the computer, and they can be minimized by automatically tuning the amplitudes and phases of the currents in one compressor with respect to those in the other.

Thermodynamic aspects of the heart scanner

The important thermodynamic parameters in the design of the conductive-strip interface are the cool-down time and the tip temperature. Both should be minimized. However,

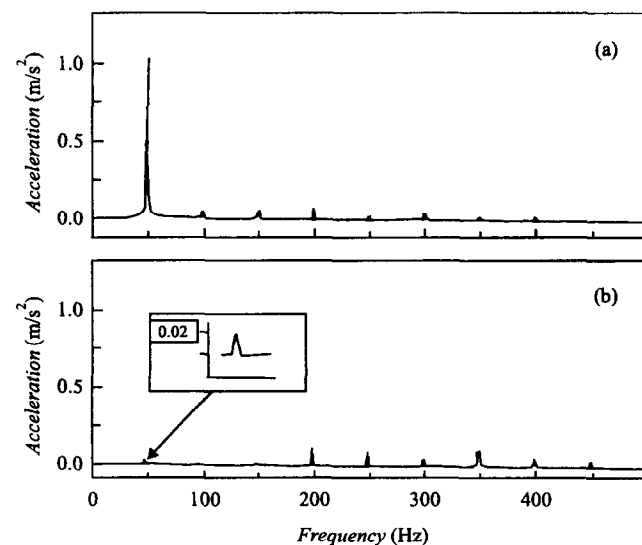


Figure 6 Vibration reduction of a single compressor by tuning piston-coil currents: before tuning (a) and after tuning (b)

a compromise has to be found since these parameters are competing: a lower cool-down time requires — for a fixed strip length — less strip material and, therefore, results in a larger temperature profile along the strip and hence a higher tip temperature.

Another important aspect in the design is the configuration of the two UP 7058 coolers. The two can be connected in parallel to cool the strip, or they can be split: one for the strip and one to cool a thermal shield. In order to evaluate the cool-down time, the tip temperature and the effectiveness of a shield configuration, a thermodynamic model is developed and numerical calculations are performed. Based on these simulations a configuration without a cooled shield has been chosen. Furthermore, more detailed calculations on this configuration show an optimum cross-sectional area of the conductive strip with respect to the cool-down time and the tip temperature. After that, measurements on a copper strip connected to a UP 7058 cooler are compared with simulation results. Finally, the suppression of small temperature oscillations generated by the cooler cold heads is considered.

Modelling the conductive strip interface

If a conductive strip is applied as a thermal interface, this strip should be at least partly flexible to minimize the mechanical coupling between the cold tips and the SQUIDs. The sensors will be mounted on a plate connected to the conducting interface. This plate should have a good thermal conductivity combined with a small electrical conductivity (necessary to limit Johnson noise). To simulate the thermodynamics of the configurations with and without a shield, a model was developed containing the following parameters:

1. The cooling power of the UP 7058 cryocooler as a function of the cold-tip temperature. In the simulations the curves of the cooling power as given in *Figure 4* are used.
2. The heat load from the environment. The cold part of the heart scanner is thermally insulated from the environment by placing it in a vacuum box. Furthermore, it will be wrapped in multilayer insulation (MLI). The environmental heat load consists of heat flow through spacers and through wires connecting the SQUIDs to the read-out electronics, and heat flow through the MLI blanket. The former flow is expressed via standard heat-conductivity integrals and the latter is determined via reference 23.
3. The thermal interface and the SQUID plate: their dimensions, heat conductivities and heat capacities. These dimensions and material properties of the thermal interface and the mounting plate are variables in the model to be optimized.

Thermodynamics of the conductive strip. The one-dimensional heat diffusion equation for a conductive strip as shown in *Figure 7* can be written (in W m^{-1}) as:

$$\rho A_c c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(\lambda A_c \frac{\partial T}{\partial x} \right) + \dot{q}'_{\text{in}}(T, T_{\text{sur}}) \quad (1)$$

Here, ρ is the specific density of the conducting material, c_p the specific heat capacity, λ the thermal conductivity, A_c the cross-sectional area, T the temperature, and x and t are

the position coordinate and time, respectively. The heat-load term $\dot{q}'_{\text{in}}$ (W m^{-1}) from the surroundings at temperature T_{sur} may be due to radiation, to heat transfer through the MLI (used, for example, around the strip) and to heat conduction from the supporting material and the leads to the SQUIDs on the SQUID plate. By definition, it holds that $\dot{q}'_{\text{in}} = d\dot{q}_{\text{in}}/dx$.

We will discuss the different contributions to $\dot{q}_{\text{in}}$. The heat transfer rate due to radiation $\dot{q}_{\text{rad}}$ for a two-surface enclosure is written as²⁴:

$$\dot{q}_{\text{rad}} = \frac{A_s \sigma (T_{\text{sur}}^4 - T^4)}{\epsilon_s^{-1} + (\epsilon_{\text{sur}}^{-1} - 1) \left(\frac{A_s}{A_{\text{sur}}} \right)} \quad (2)$$

in which A_s is the outer surface area of the enclosed conducting material having an emission coefficient ϵ_s , A_{sur} is the inner surface area of the enclosing body with an emission coefficient ϵ_{sur} , and σ is the Stefan-Boltzmann constant equal to $5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$. For infinitely long concentric cylinders, we have $A_s/A_{\text{sur}} = R_s/R_{\text{sur}}$, in which R_s is the outer radius of the enclosed cylinder, i.e. the conducting strip, and R_{sur} is the inner radius of the enclosing cylinder (e.g. a radiation shield). For infinitely large parallel planes, it holds that $A_s/A_{\text{sur}} = 1$ in Equation (2). In the model, the strip and the enclosing radiation shield are considered as infinitely long concentric cylinders, and the SQUID plate and its enclosing shield as infinitely large parallel planes.

The heat transfer rate $\dot{q}_{\text{MLI}}$ through MLI can be expressed as²³:

$$\dot{q}_{\text{MLI}} = k_{\text{MLI}} A \frac{(T_N - T_1)}{\Delta d} \cdot \left(\frac{N}{N-1} \right) \quad (3)$$

in which k_{MLI} is a mean effective heat transfer coefficient that includes radiation and conduction through the MLI ($\text{W m}^{-1} \text{ K}^{-1}$), and A is the area through which the heat is transferred; N is the number of layers of superinsulation that are placed with an overall thickness of Δd between a cold and a hot surface; and T_1 (T_N) is the temperature of the first layer at the cold (hot) surface side. Ter Brake and Flokstra²³ give a minimum value for $k_{\text{MLI}}^{\text{min}}$ ($k_{\text{MLI}}^{\text{min}} = k_{\text{MLI}}/\Delta d$) of $4 \times 10^{-3} \text{ W m}^{-2} \text{ K}^{-1}$ at an optimum layer density of 25–30 layers per cm for $T_1 = 77 \text{ K}$ and $T_N = 300 \text{ K}$. These optimum values will be used in all further considerations, assuming that the optimum layer density can be established. If it is also assumed that the temperatures T_1 and T_N are equal or close to T and T_{sur} , respectively, Equation (3) can be approximated by:

$$\dot{q}_{\text{MLI}} = k_{\text{MLI}}^{\text{min}} A (T_{\text{sur}} - T) \quad (4)$$

The heat transfer rate $\dot{q}_{\text{cond}}$ due to the heat conduction from the supporting material and the leads to the SQUIDs can be written as:

$$\dot{q}_{\text{cond}} = \frac{A_{c,ls}}{\Delta L_{ls}} \int_T^{T_{\text{sur}}} \lambda_{ls}(T) dT \quad (5)$$

where λ_{ls} and $A_{c,ls}$ are the thermal conductivity and the cross-sectional area, respectively, of the leads or the sup-

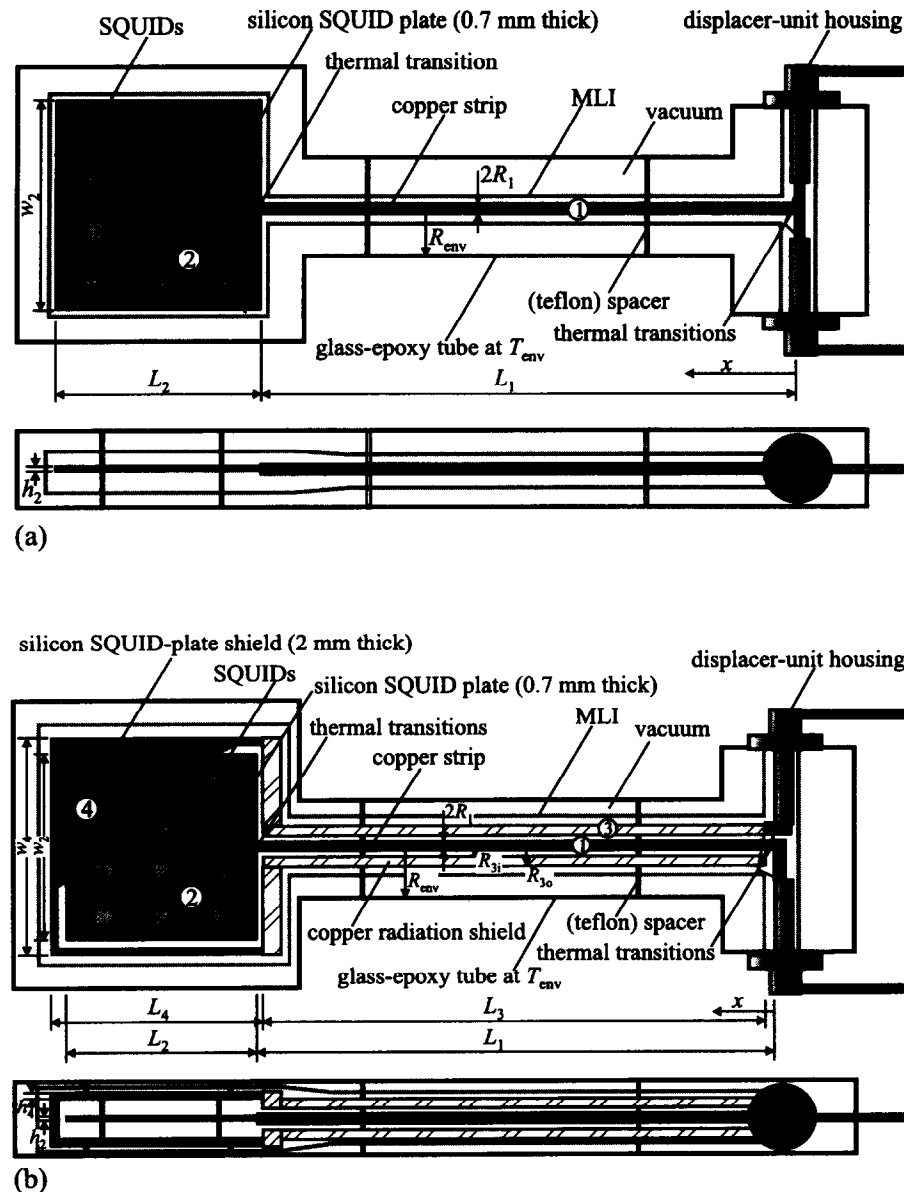


Figure 7 Schematic models of conductive-strip interface between the two UP 7058 coolers and the SQUID plate: configurations without (a) and with (b) a cooled shield. The numbers refer to the indices used in the text

porting material, and ΔL_{is} is the length of the leads or supporting material.

Thermodynamics of two conductive strip configurations: assumptions, equations and choices. The following assumptions are made in the thermodynamic model:

1. In the conductive strip and its cylindrical shield the temperature is uniform in the radial direction. The temperature distribution in the SQUID plate (and its shield) is uniform in the directions normal to the x -axis. If such a uniform temperature distribution over the cross-sectional areas of the configuration elements exists, a one-dimensional approach (e.g. in the x -direction as in Equation (1)) is valid. A uniform cross-section temperature distribution will be established if the dimensionless Biot number²⁴ $Bi = kL_c/\lambda \ll 1$, where λ ($\text{W m}^{-1} \text{K}^{-1}$) represents the thermal conductivity of the solid material, L_c is a characteristic length normal to

the x -axis for the solid material (e.g. for the strip $L_c = R_1$) and k ($\text{W m}^{-2} \text{K}^{-1}$) represents the conduction from the solid material to the surroundings (see Figure 7).

2. The heat capacity of the cooler is neglected, which is a reasonable assumption since the heat capacity of the cooler is small compared with the heat capacity of the configuration elements connected to the cooler.
3. The conductive heat leak (Equation (5)) due to leads to the SQUID plate and its surrounding shield is uniformly distributed over this plate and its shield. This heat load is much larger than the contribution due to spacers. Therefore, the heat load from the latter is neglected. The conductive heat leak to the strip and its shield is due to only two spacers, one at the beginning of the strip and shield and one at the end.
4. The material densities are considered to be temperature independent.
5. The heat leakage due to radiation onto the SQUID plate unit from the side walls of the box is negligible com-

pared with the contribution from the planes parallel to the SQUID plate.

6. The heat capacities of the applied MLI, leads and spacers are negligible compared to the capacity of the other elements of the configuration.
7. The thermal resistance of the transitions between the strip (and its shield) and the SQUID plate (and its shield) are temperature independent.

Two different configurations will be considered separately: one without and one with a cooled shield.

Without cooled shield The configuration of a conductive strip without a cooled shield is depicted in *Figure 7a*. The strip is placed in a vacuum space, wrapped in MLI, and connected to and cooled by both coolers. Moreover, a heat conduction contribution $\dot{q}'_{ls}$ (with $\dot{q}'_{ls} = d\dot{q}_{ls}/dx$) due to the leads and the supporting material is present. The model for the configuration is divided into two parts: one for the strip, the other one for the SQUID plate. In this case the partial differential equation for the strip follows from manipulating Equations (1) and (4):

$$\rho_1 A_{c,1} c_{p,1} \frac{\partial T_1}{\partial t} = \frac{\partial}{\partial x} \left(\lambda_1 A_{c,1} \frac{\partial T_1}{\partial x} \right) + k_{MLI}^{min} \cdot 2\pi R_1 (T_{env} - T_1) + \dot{q}'_{ls,1}$$

where the index 1 refers to the strip, and T_{env} is the environmental temperature of the enclosure around the strip. As shown in *Figure 7a*, the SQUID plate is connected to the end of the strip via a thermal transition resistance. This plate is also placed in a vacuum space and wrapped in MLI. Again, a heat conduction contribution $\dot{q}'_{ls}$ due to the leads and the supporting material is present. The equation for the SQUID plate can be written as:

$$\rho_2 w_2 h_2 c_{p,2} \frac{\partial T_2}{\partial t} = \frac{\partial}{\partial x} \left(\lambda_2 w_2 h_2 \frac{\partial T_2}{\partial x} \right) + k_{MLI}^{min} \cdot 2w_2 (T_{env} - T_2) + \dot{q}'_{ls,2} \quad (7)$$

in which the index 2 refers to the SQUID plate, w is the width and h the thickness of the plate. At the transition a boundary condition that connects Equations (6) and (7) is satisfied.

With cooled shield The configuration of a conductive strip with a cooled thermal shield is schematically shown in *Figure 7b*. One cooler cools the strip and the other cools the shield. Again a SQUID plate is connected to the strip via a thermal transition resistance and both are surrounded by a thermal shield. All configuration elements are placed in a vacuum surrounded by an enclosure at the environmental temperature T_{env} . Between the shield and the enclosure at T_{env} several layers of MLI are used. The equation for the strip (referred to by index 1) and for the shield that surrounds this strip (index 3) can be written as:

$$\rho_1 A_{c,1} c_{p,1} \frac{\partial T_1}{\partial t} = \frac{\partial}{\partial x} \left(\lambda_1 A_{c,1} \frac{\partial T_1}{\partial x} \right) + \frac{2\pi R_1 \sigma}{\epsilon_1^{-1} + \left(\frac{R_1}{R_{3i}} \right) (\epsilon_3^{-1} - 1)} (T_3^4 - T_1^4) + \dot{q}'_{ls,1} \quad (8)$$

and

$$\rho_3 A_{c,3} c_{p,3} \frac{\partial T_3}{\partial t} = \frac{\partial}{\partial x} \left(\lambda_3 A_{c,3} \frac{\partial T_3}{\partial x} \right) + \frac{-2\pi R_1 \sigma (T_3^4 - T_1^4)}{\epsilon_1^{-1} + \left(\frac{R_1}{R_{3i}} \right) (\epsilon_3^{-1} - 1)} + k_{MLI}^{min} P_3 (T_{env} - T_3) - \dot{q}'_{ls,1} + \dot{q}'_{ls,3e} \quad (9)$$

respectively, where R_{3i} is the inner radius of the shield and P_3 is the outer perimeter of the cylindrical shield which is equal to πR_{3o} (R_{3o} is the outer radius of the cylindrical shield). For the SQUID plate (index 2) and its surrounding shield (index 4) the equations are written as:

$$\rho_2 w_2 h_2 c_{p,2} \frac{\partial T_2}{\partial t} = \frac{\partial}{\partial x} \left(\lambda_2 w_2 h_2 \frac{\partial T_2}{\partial x} \right) + \frac{2w_2 \sigma (T_4^4 - T_2^4)}{\epsilon_2^{-1} + \epsilon_4^{-1} - 1} + \dot{q}'_{ls,2} \quad (10)$$

and

$$\rho_4 w_4 h_4 c_{p,4} \frac{\partial T_4}{\partial t} = \frac{\partial}{\partial x} \left(\lambda_4 w_4 h_4 \frac{\partial T_4}{\partial x} \right) + \frac{-2w_2 \sigma (T_4^4 - T_2^4)}{\epsilon_2^{-1} + \epsilon_4^{-1} - 1} + k_{MLI}^{min} 2w_2 (T_{env} - T_4) - \dot{q}'_{ls,2} + \dot{q}'_{ls,4} \quad (11)$$

respectively. The factor 2 in these latter two equations comes from the two parts of the shield at both sides of the SQUID plate.

Boundary and initial conditions. For solving Equations (6) and (7) of the configuration without a cooled shield (configuration A), and Equations (8)–(11) of the configuration with a cooled shield (configuration B), the following boundary and initial conditions are fulfilled:

$$\begin{aligned} \lambda_i A_{c,i} \frac{\partial T_i}{\partial x} \Big|_{x=0} &= n \dot{q}_{cc} (T_i(x=0)), \text{ with } (i=1,3) \\ \lambda_i A_{c,i} \frac{\partial T_i}{\partial x} \Big|_{x=L_i} &= \lambda_{i+1} A_{c,i+1} \frac{\partial T_{i+1}}{\partial x} \Big|_{x=L_i}, \text{ with } (i=1,3) \\ \frac{\partial T_{i+1}}{\partial x} \Big|_{x=L_i+L_{i+1}} &= 0, \text{ with } (i=1,3) \\ T_j(x,t=0) &= T_{env}, \text{ with } (j=1 \dots 4) \end{aligned} \quad (12)$$

Here, n depends on the number of coolers applied to cool the configuration. For configuration A, it holds that $n=2$ and $i=1$; for configuration B, it holds that $n=1$ and $i=1$, and $n=1$ and $i=3$. The first two boundary conditions in Equation (12) indicate the continuity of the heat flux on the transition from the coolers to the strip or shield, and on that from the strip (and its shield) to the SQUID plate (and its surrounding shield).

Material choices and properties. As discussed in the section on cooler interference, in the model $L_1 = L_3 = 0.3$ m. For multichannel measurements several SQUIDs need to be incorporated, all carried by the SQUID plate. Since the magnetometer system is designed for 25 sensors, the plate

should be large enough to bear all with a baseline distance³ of a few cm. Therefore, the dimensions chosen are $L_2 = 0.15$ m and $w_2 = 0.15$ m. In the calculations, the surrounding shield is assumed to have the same dimensions: $L_4 = w_4 = 0.15$ m. The cooling power of a single UP 7058 cooler is shown in Figure 4 (curve a). With the cold finger placed in a vacuum space and without additional thermal load to the tip, this cooling power can be established within a few minutes¹¹.

As the material for the strip and its shield we intend to use copper. The heat conductivity λ_{Cu} ²⁵ and heat capacity $c_{p,Cu}$ ²⁶ are shown as a function of temperature in Figure 8a. For the SQUID plate and its shield silicon is considered, because of its low Johnson noise level at the operating temperature (due to the high exponential decrease of the number of free charge carriers with decreasing temperature) and a relatively high thermal conductivity. The thickness h_2 of the silicon SQUID plate is 0.7 mm, the thickness of an available silicon 8-inch wafer (with a measured resistivity $\rho_4 \approx 0.1$ k Ω m at room temperature), whereas the thickness h_4 for the shield was chosen at 2 mm. The thermal conductivity and heat capacity for silicon, λ_{Si} ²⁷ and $c_{p,Si}$ ²⁸, respectively, are shown in Figure 8b. Further, copper has a density ρ_{Cu} of 8.91×10^3 kg m⁻³, and silicon a density ρ_{Si} of 2.33×10^3 kg m⁻³²⁹. For the surrounding enclosure at T_{env} glass-epoxy is suitable, since this material gives no Johnson noise contribution.

In order to evaluate the heat leakages, it is assumed that in both configurations the inner radius R_{env} of the cylinder at T_{env} is kept constant at 11 mm. The distance between the SQUID plate and its surrounding shield is fixed at 6 mm. If a shield is installed, the inner radius of the shield is also fixed constant at 6 mm. The system is designed for 25 SQUIDs each requiring eight connections to the outside electronics. Manganin wire MW-36 of Lake Shore Cryotronics³⁰ (with a wire diameter of 0.127 mm and a resistance of ~ 10 Ω m⁻¹) is considered as an example, having a heat-conductivity integral $\int \lambda_{MW} dT$ ³¹ as shown in Figure 8c. It is assumed that 0.1 m long lines can be installed; if a shield is used these leads are anchored half-way to the shield. Further, teflon spacers will be used each with an assumed thickness of 1 mm, ΔL_{ts} of maximum 5 mm and a cross-sectional area of roughly 11–69 m², depending on whether or not a shield is used and on the cross-sectional areas of the strip and/or shield. The heat-conductivity integral $\int \lambda_{teflon} dT$ of teflon³² is shown in Figure 8d. The thickness Δd of the layers of MLI in both configurations is about 5 mm. All emission coefficients are assumed to be 0.04, a value for polished copper³³.

Based on these material choices and their material properties, the Biot number can be evaluated. If both the strip and the SQUID plate are wrapped in 0.5 cm MLI, the resulting effective heat transfer coefficient k_{MLI} is 8×10^{-3} W m⁻¹ K⁻¹. Hence, even in the worst case situations (thermal conductivities at 300 K), for both the strip and the SQUID plate and the shields the Biot number is sufficiently less than 1. Therefore, the assumption of a uniform cross-sectional temperature distribution in the strip and its shield and the SQUID plate and its shield is valid, and the temperature distribution is essentially one-dimensional.

Numerical simulations. Numerical simulations have been performed on the configurations with and without a cooled shield. These simulations are based on the implicit

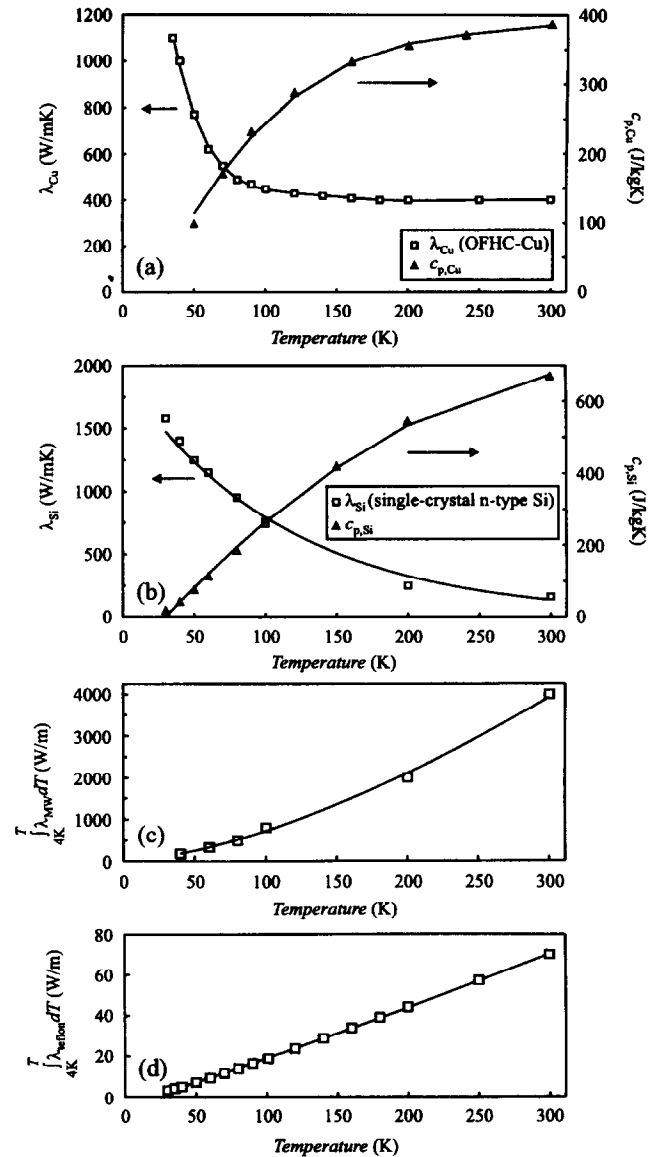


Figure 8 Heat conductivities λ and heat capacities c_p versus temperature T of copper^{25,26} with an RRR value of ≈ 100 (a) and of silicon^{27,28} (b); heat-conductivity integral $\int \lambda dT$ of manganin wire³¹ (c), and of teflon³² (d)

Crank–Nicolson (CN) differencing scheme (Δx is the grid size and Δt the period of one time-step in the simulation). The radiative heat flow is linearized by the Newton–Raphson method.³⁴ The CN method assures a high accuracy (a second-order truncation error in Δx and Δt) combined with an inherent stability for any magnitude of Δx and Δt .

The grid size Δx in the strip, in the SQUID plate and in the shields was 1 cm. The period of one time-step Δt was usually 0.1 s. The temperature error resulting from these discrete steps depends on the type of heat load and is estimated to be 0.1 K for conductive flow (via MLI) and 0.01 K for radiative flow.

Optimization and comparison of both configurations: results

Using numerical simulations, both configurations (with and without a cooled shield) were optimized with respect to the temperature of the SQUID plate and the cool-down time of

the cooling system. In addition, the results for both configurations are compared. During cool-down, a drift occurs in the output of the SQUIDs due to their temperature decrease. In the section on design goals and constraints, the maximum acceptable temperature fluctuation was set to 2 mK_{pp} . For the lowest frequency in the measuring band, i.e. 0.1 Hz , this corresponds to a transient of 0.6 mK s^{-1} . Related to this value we defined the cool-down time τ_{cd} of the cooling system as the time needed to establish a temperature stability of 0.1 mK s^{-1} at the centre of the SQUID plate. In the simulations, the diameters of the strip and the shield as well as the heat leakage due to the leads have been varied.

Figure 9 shows simulation results of the configuration without a cooled shield. It shows the tip temperature T_{tip} at the end of the silicon SQUID plate versus the cool-down time τ_{cd} for varying cross-sectional areas $A_{c,1}$ of the copper strip and for different heat loads to the SQUID plate. In the figure, $A_{c,1}$ decreases along the curves in the direction indicated by arrows; in the first two 'steps' (a 'step' indicates a new simulation run) $A_{c,1}$ decreases from $1 \times 10^{-4} \text{ m}^2$ with steps of $2 \times 10^{-5} \text{ m}^2$, in the following with steps of $5 \times 10^{-6} \text{ m}^2$ down to $1 \times 10^{-5} \text{ m}^2$. The heat load of this configuration is varied by the number n_{leads} of the leads to the SQUID plate, thus demonstrating the sensitivity of the results on the heat load. The 200, 400 and 800 leads correspond roughly to a heat load to the plate in the final stationary state of 0.2, 0.3 and 0.5 W, respectively, including about 0.1 W due to the heat load from the MLI; the heat load due to spacing the strip to avoid thermal contact) appeared to be much smaller than the heat load due to the MLI, and the heat load due to radiation appeared to be negligible small. (For wiring of 25 SQUIDs, 200 leads could be required.)

Figure 9 shows a minimum cool-down time corresponding to an optimum cross-sectional area $A_{c,1,opt}$ of $3 \times 10^{-5} \text{ m}^2$ for the different heat loads. T_{tip} corresponding to these minimum cool-down times increases with increasing heat load. With a heat load of 0.3 W an optimum T_{tip} at the end of the SQUID plate of 50 K is reached in 64 min. Since the optimum tip temperature is rather sensitive for small changes in the cross-sectional area for $A_{c,1} < A_{c,1,opt}$, for the design we chose a value slightly above $A_{c,1,opt}$:

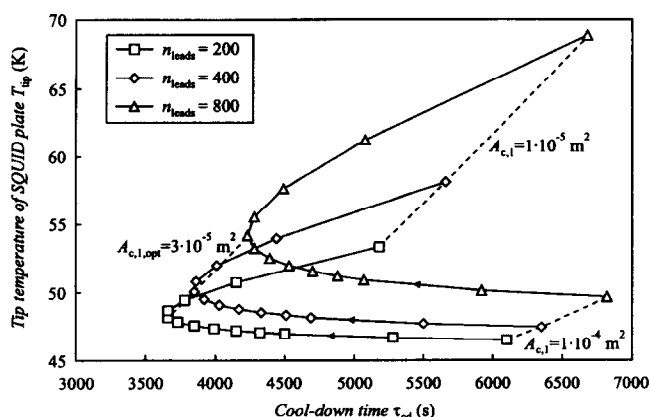


Figure 9 Simulation results for configuration without a cooled shield: tip temperature T_{tip} at end of silicon SQUID plate versus cool-down time τ_{cd} for varying cross-sectional areas $A_{c,1}$ of copper strip and for different heat loads to SQUID plate. The heat load is varied through the number n_{leads} to the SQUID plate (see text). $A_{c,1}$ decreases along the curves in the direction indicated by the arrows

$4 \times 10^{-5} \text{ m}^2$, corresponding to a diameter of 7 mm. For a heat load of 0.3 W this corresponds to a T_{tip} at the end of the SQUID plate of 49 K in 67 min.

For $A_{c,1} > A_{c,1,opt}$, the cool-down time τ_{cd} and T_{tip} are mainly determined by the heat capacity (or mass) of the copper strip. Then, with decreasing $A_{c,1}$ the thermal resistance of the copper strip increases and its heat capacity decreases, resulting in a higher T_{tip} — due to a larger temperature profile along the copper strip — and a shorter τ_{cd} . For $A_{c,1} < A_{c,1,opt}$, the τ_{cd} and T_{tip} are mainly determined by the heat capacity of the silicon plate and the thermal resistance of the copper strip. Then, with increasing $A_{c,1}$ the resistance of the copper strip decreases at the expense of an increase in its capacity, resulting in a lower T_{tip} and, again, a shorter τ_{cd} . As a result, an optimum value for the cool-down time exists in between.

Figure 10 shows the simulation results of the configuration with a cooled shield. It shows both the tip temperature T_{tip} at the end of the SQUID plate and the tip temperature T_{shield} at the end of the shield of the SQUID plate versus the cool-down time τ_{cd} . In the figure, the heat loads to the SQUID plate and its shield are varied via the number of leads, similarly as described above. Also the cross-sectional areas of the strip $A_{c,1}$ and its surrounding shield $A_{c,3}$ are varied, with $A_{c,1} = A_{c,3}$; $A_{c,1}$ and $A_{c,3}$ decrease again along the curves from $1 \times 10^{-4} \text{ m}^2$ down to $1 \times 10^{-5} \text{ m}^2$ in the direction indicated by arrows with the same step-sizes as described above.

Again, minimum cool-down times are observed for optimum cross-sections $A_{c,1,opt} = A_{c,3,opt} = 3 \times 10^{-5} \text{ m}^2$. Both tip temperatures (T_{tip} and T_{shield}) corresponding to these minimum cool-down times increase with increasing heat load. The minimum cool-down times are roughly twice as large as the values found from the configuration without a cooled shield (Figure 9), caused by the larger mass in the configuration with a shield. For instance, for a heat load of about 0.3 W, $A_{c,1,opt} = A_{c,3,opt} = 3 \times 10^{-5} \text{ m}^2$ corresponds to a cool-down time of 125 min; then, $T_{tip} = 45.3 \text{ K}$ and $T_{shield} = 62 \text{ K}$. This low T_{tip} value is caused by the thermal shielding that encloses the strip and the SQUID plate. Moreover, the manganin wires are anchored on the SQUID-

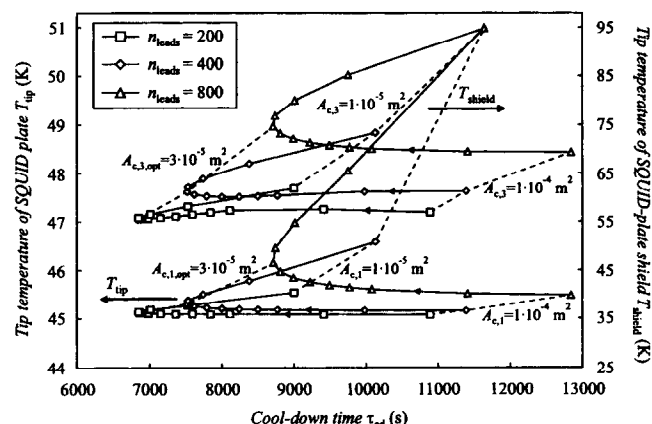


Figure 10 Simulation results for configuration with a cooled shield: tip temperature T_{tip} at end of SQUID plate and tip temperature T_{shield} at end of SQUID plate shield versus cool-down time τ_{cd} . The heat load to the SQUID plate and its shield is varied via the number n_{leads} ; the cross-sectional areas of the strip $A_{c,1}$ and its surrounding shield $A_{c,3}$ decrease along the curves in the direction indicated by arrows (see text). In the simulations, $A_{c,1} = A_{c,3}$

plate shield reducing the conductive heat load on the SQUID plate.

In comparison, in the configuration with a shield the minimum cool-down time depends more strongly on the heat load, because each single cooler has to cool the same or even a higher heat capacity than both coolers have to in the simpler configuration without a shield. In the configuration with a shield, T_{tip} at the end of the SQUID plate increases less fast with increasing heat load than in the configuration without a shield. This is caused by the enclosing thermal shield around both the strip and the SQUID plate, and the thermal anchoring of the wires to the SQUID plate on the SQUID-plate shielding, thus reducing the heat load from wiring to the SQUID plate.

For $n_{leads} = 200$ and $n_{leads} = 400$, T_{shield} decreases slightly with decreasing values of $A_{c,3}$ from $A_{c,3} = 1 \times 10^{-4} \text{ m}^2$. This mainly originates from a decrease in the heat load due to the MLI which is assumed to be proportional to the outer area of the shield.

In conclusion, the cool-down time for the configuration with a cooled shield is roughly twice as large as the time for the configuration without a cooled shield. On the other hand, in the configuration with a cooled shield, the temperature T_{tip} at the end of the SQUID is lower and less sensitive to heat load variations (resulting in a more stable T_{tip}) than in the other configuration. Moreover, in further development of the model for the configuration with a cooled shield, the distribution of the cooling power between the shield and the strip can be optimized; then, a shorter τ_{cd} can be obtained. However, with respect to the required temperature of the SQUIDs (see the section Design goals and constraints), the obtainable tip temperature for the configuration without a shield is acceptable. With a heat load of 0.3 W, for instance, at the optimum cross-sectional area $A_{c,1,opt}$ of $3 \times 10^{-5} \text{ m}^2$ a T_{tip} value of 50 K can be attained in 64 min. Besides the more rapid cool-down to an acceptable low tip temperature, its simpler construction is also an advantage for the configuration without a shield. Therefore, this latter configuration is selected for realization.

Measurements on the conductive-strip model

The developed thermodynamic model has been validated by experiments. In a test set-up (Figure 11) a 7 mm diameter copper rod of 30 cm in length was connected to a single cold head via a 13 cm long flexible copper link (with an effective cross-sectional area of 40 mm^2). Five diodes were mounted on the arrangement as thermometers³⁰. Diode 1 measured the temperature of the cold tip, whereas diodes 2–5 measured the temperature profile along the rod. The rod was wrapped in 15 layers of MLI and placed in a vacuum can (see Figure 12). Black-coated cooling fins of total area 2000 cm^2 and 640 cm^2 were placed on the compressor and displacer housings, respectively. A cool-down registration is depicted in Figure 13, where the temperatures as recorded by the diodes are given along with the simulation results. As an input parameter in the simulation we did not directly use the specified cooling power, but instead we measured it. (The cooling power depends on the input power and the heat contact of the compressor and the displacer housings to the environment to heat sink the heat of compression.) We measured a cooling power $\dot{q}_{exp}$ that depends linearly on the cold-tip temperature T as $\dot{q}_{exp} = aT - b$, with $a = 0.04 \text{ W K}^{-1}$ and $b = 1.73 \text{ W}$. In the measurements the input power was 53 W and the tempera-

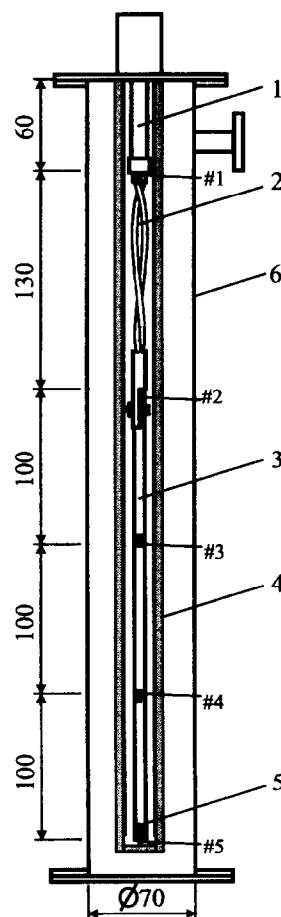


Figure 11 Test set-up (schematic): cold finger (1), flexible link (2), copper rod (3), multilayer insulation (4), heater (5) and vacuum can (6). The positions of temperature diodes 1–5 are also indicated

ture of the cold-head housing 50°C . Corrected for both the input power and the housing temperature, this measured $\dot{q}_{exp}$ agrees with the curve specified by Signaal Usfa (curve b in Figure 4). In the simulation the thermal conductivity of the copper materials was chosen based on the measured RRR values (RRR is the ratio between the resistivity at room temperature and that in liquid helium, at 4.2K)³⁵. From the results presented in Figure 13, it is concluded that the model describes the thermodynamic behaviour of the test set-up very well. Therefore, it is used to design the cryogenic part of the heat scanner.

Suppression of temperature waves generated at the cryocooler cold head

The coolers are operated with driving currents at a frequency of 50 Hz. In the cold head helium gas will also expand this frequency, causing a small harmonic temperature fluctuation superimposed on the time-averaged tip temperature T_{mean} . Because the conductive strip is connected to the tip, temperature fluctuations with amplitude A_0 will propagate as temperature waves into the strip. Solving Equation (1) — but without any heat load or loss ($\dot{q}'_{in} = 0$) — for locally damped, time-periodic functions, the temperature can be expressed as:

$$T(x,t) = T_{mean} + A_0 e^{-\sqrt{\frac{\omega}{2a}}x} e^{i\omega(t - \frac{x}{\sqrt{2a\omega}})}, \quad a = \frac{\lambda}{\rho c_p} \quad (13)$$

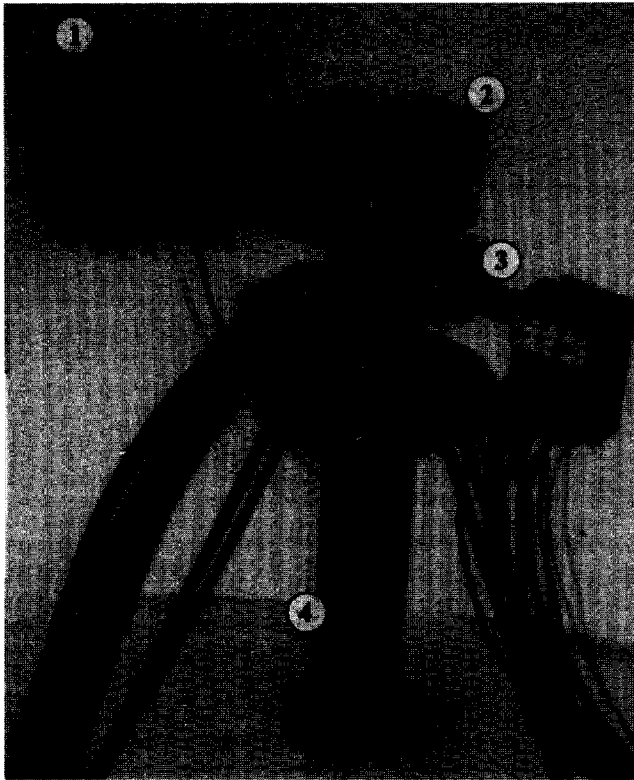


Figure 12 Photograph of test set-up: compressor (1), cold head (2), data port (3) and vacuum can (4)

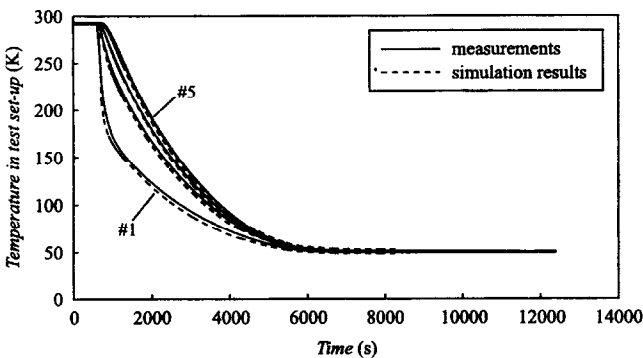


Figure 13 Cool-down registration of test set-up

where t is the time, x the position along the strip, ω the radial frequency of the temperature fluctuation, λ the heat conductivity, ρ the mass density and c_p the heat capacity of copper. In our coolers, cold-tip temperature fluctuations at 50 Hz are not expected¹² to exceed 0.1 K. From Equation (13) it can be derived that for a frequency of 50 Hz and a typical strip temperature of 50 K, the amplitude of the temperature fluctuation decreases by a factor of about 5×10^3 over a distance of 2 cm. Therefore, using a strip with a length L of 0.3 m, effects of temperature waves generated at the cold head are negligible at the end of the strip.

Conclusion: cryogenic design of the heart scanner

A refrigeration system was designed in which commercially available cryocoolers are used for cooling a heart scanner equipped with 25 high- T_c SQUID magnetometers. In the heart scanner the SQUIDs will be mounted on a

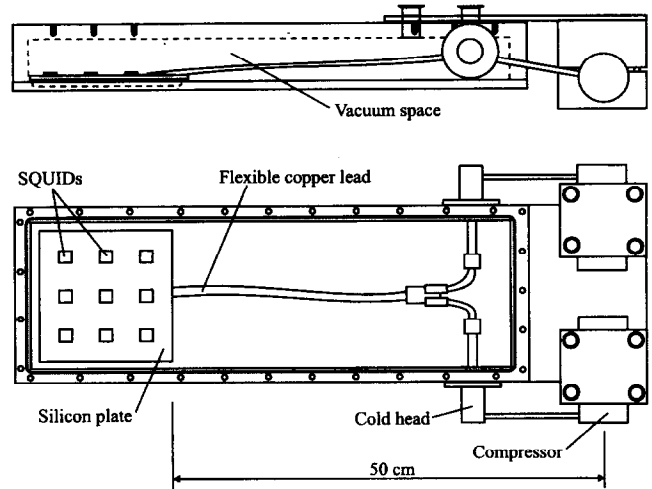


Figure 14 Heart scanner design (schematic), depicted with nine SQUIDs

15 cm $\times$ 15 cm $\times$ 0.7 mm silicon SQUID plate which is cooled by two Signaal Usfa UP 7058 Stirling coolers. The cooler compressors are placed 0.5 m from the pick-up coils of the SQUIDs in order to limit the magnetic interference. This interference is further minimized by positioning the compressors coplanar with the SQUID pick-up coils. The vibrations of the coolers are reduced by mounting them in a back-to-back configuration, and operating them in counterphase. The cold heads of the coolers are thermally linked to the SQUID plate by two flexible copper leads (i.e. two Litze cables consisting of separate twisted and non-insulated copper wires, each cable with an effective cross-sectional area of 20 mm² and a length of 30 cm). The SQUID plate is cut from an 8-inch silicon wafer. The copper leads will be connected to the silicon SQUID plate by splitting the Litze cables at the plate's end into separate wires, and soldering these wires onto copper pads deposited on the silicon plate. Together with the thermal leads the SQUID plate will be wrapped in about 15 layers MLI and placed in a rectangular non-metallic (e.g. glass-epoxy) vacuum box. The coolers are mounted on this box in a rigid manner as discussed before in the section on cooler interference. The SQUIDs will be glued onto the silicon plate and the electrical connections to the outside world will be made with 0.1 m long manganin high-resistance wires to reduce the thermal load. Teflon spacers are used for support of the copper leads and the SQUID plate inside the vacuum box. The design is schematically depicted in Figure 14. A theoretical cool-down curve for this heart scanner design, when equipped with 25 SQUIDs, is presented in Figure 15. Here,

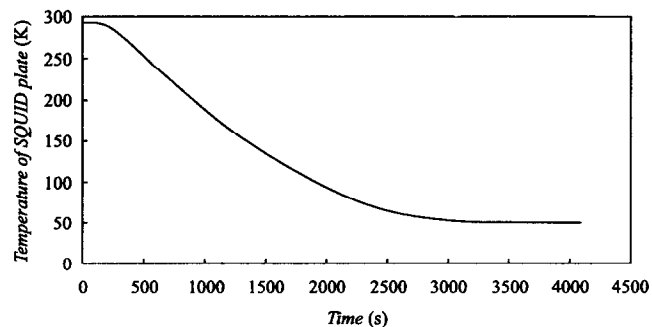


Figure 15 Calculated cool-down curve of heart scanner design (with 25 SQUIDs): temperature of SQUID plate versus time

twice the cooling power as specified in Figure 4 (curve b) is applied. In this thermodynamic simulation no additional heat capacity was taken into account for SQUID packaging. In this case, a SQUID-plate temperature below 55 K is expected in about 50 min. The tip temperature of the SQUID plate can be controlled by the application of a heater at the SQUID plate in a feedback circuit; this decreases the cool-down time of the cooling system at the expense of a few K higher T_{tip} . Moreover, a more stable final tip temperature can be achieved, because more cooling power of the cooler is available at a higher temperature.

Acknowledgements

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